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Tekst 19: Pascal om hasardspil

Flere italienske matematikere havde før 1600 diskuteret problemet om, hvordan man skal dele puljen mellem to hasardspillere, som bliver afbrudt i deres spil. Formentlig i 1654 fremlagde Chevalier de Méré, der selv var ivrig spiller, problemet for Pascal og Fermat. Problemet — nedenfor kaldet the problem of the points og på dansk som regel kaldet delingsproblemet — går ud på følgende: To personer spiller et lige spil, fx “plat og krone”. Den der først vinder et på forhånd aftalt antal spil, n, vinder puljen. Spillerne bliver imidlertid afbrudt før spillet er færdig. Hvordan skal de så dele puljen? Pascal fremsatte sit forslag til en løsning på problemet i et brev til Fermat fra 1654. Nedenfor er gengivet den engelske oversættelse i [Smith 1959, pp. 547–49]. Puljen er her på 64 pistoles (en fransk møntenhed fra 1600-tallet).

Pascal to Fermat, Wednesday, July 29, 1654

Monsieur,—

1. Impatience has seized me as well as it has you, and although I am still abed, I cannot refrain from telling you that I received your letter in regard to the problem of the points1 yesterday evening from the hands of M. Carcavi, and that I admire it more than I can tell you. I do not have the leisure to write at length, but, in a word, you have found the two divisions of the points and of the dice with perfect justice. I am thoroughly satisfied as I can no longer doubt that I was wrong, seeing the admirable accord in which I find myself with you. I admire your method for the problem of the points even more than that of the dice. I have seen solutions of the problem of the dice by several persons, as M. le chevalier de Méré, who proposed the question to me, and by M. Roberval 1The editors of the letter notes that the word parti means the division of the stake between the players in the case when the game is abandoned before its completion. Parti des d´es means that the man who holds the die agrees to throw a certain number in a given number of trials. For clarity, in this translation, the first of these cases will be called the problem of the points, a term which has had a certain acceptance in the histories of mathematics, while the second may by analogy be called the problem of the dice. also. M. de Méré has never been able to find the just value of the problem of the points nor has he been able to find a method of deriving it, so that I found myself the only one who knew this proportion.

2. Your method is very sound and it is the first one that came to my mind in these researches, but because the trouble of these combinations was excessive, I found an abridgment and indeed another method that is much shorter and more neat, which I should like to tell you here in a few words; for I should like to open my heart to you henceforth if I may, so great is the pleasure I have had in our agreement. I plainly see that the truth is the same at Toulouse and at Paris. This is the way I go about it to know the value of each of the shares when two gamblers play, for example, in three throws, and when each has put 32 pistoles at stake:

Let us suppose that the first of them has two (points) and the other one. They now play one throw of which the chances are such that if the first wins, he will win the entire wager that is at stake, that is

to say 64 pistoles. If the other wins, they will be two to two and in consequence, if they wish to separate, it follows that each will take back his wager that is to say 32 pistoles. Consider then, Monsieur, that if the first wins, 64 will belong to him. If he loses, 32 will belong to him. Then if they do not wish to play this point, and separate without doing it, the first should say "I am sure of 32 pistoles, for even a loss gives them to me. As for the 32 others, perhaps I will have them and perhaps you will have them, the risk is equal. Therefore let us divide the 32 pistoles in half, and give me the 32 of which I am certain besides." He will then have 48 pistoles and the other will have 16. Now let us suppose that the first has two points and the other none, and that they are beginning to play for a point. The chances are such that if the first wins, he will win all of the wager, 64 pistoles. If the other wins, behold they have come back to the preceding case in which the first has two points and the other one. But we have already shown that in this case 48 pistoles will belong to the one who has two points. Therefore if they do not wish to play this point, he should say, "If I win, I shall gain all, that is 64. If I lose, 48 will legitimately belong to me. Therefore give me the 48 that are certain to be mine, even if I lose, and let us divide the other 16 in half because there is as much chance that you will gain them as that I will." Thus he will have 48 and 8, which is 56 pistoles. Let us now suppose that the first has but one point and the other none. You see, Monsieur, that if they begin a new throw, the chances are such that if the first wins, he will have two points to none, and dividing by the preceding case, 56 will belong to him. If he loses, they will be point for point, and 32 pistoles will belong to him. He should therefore say, "If you do not wish to play, give me the 32 pistoles of which I am certain, and let us divide the rest of the 56 in half. From 56 take 32, and 24 remains. The divide 24 in half, you take 12 and I take 12 which with the 32 will make 44.

Opgave 1

Først i brevet omtaler Pascal problemet "the problem of the dice". Her holder en person på, at han kan få et bestemt antal øjne med to terninger i løbet af et bestemt antal kast.

- **a)** Formulér "the problem of the dice" med dine egne ord
- **b)** Hvor mange gange skal man kaste, før man med fordel kan holde på, at man slår mindst én dobbelt sekser?
- **c)** Forklar, hvorfor løsningen på problemet kan løses vha. en binomialfordelt stokastisk variabel.
- **d)** Besvar spørgsmål b) igen, men nu for "trippel 6'er" med 3 terninger.

Opgave 2

- **a)** Formulér delingsproblemet med dine egne ord og med en nutidig case.
- **b)** Gennemgå Pascals løsning.

Brevvekslingen mellem Fermat og Pascal om hasardspil angives undertiden som begyndelsen til sandsynlighedsregningen.

- **c)** Hvad siger Pascal om sandsynligheder i brevet? Hvad er det centrale begreb hos Pascal?