Leonhard Euler, *Introductio in Analysin Infinitorum*[1394]

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[104]*§ 139*. Let now in the formulas of § 133[1395] n be an *infinitely small* number, or $n = \frac{1}{i}$, i being an *infinitely large* number. Then

$$\cos nz = \cos \frac{z}{i} = 1 \quad \text{and} \quad \sin nz = \sin \frac{z}{i} = \frac{z}{i} \,.$$

The sine of a vanishing arc $\frac{z}{i}$ is indeed equal to the arc itself, whereas the cosine = 1. On that foundation one has

$$1 = \frac{(\cos z + \sqrt{-1}\,\sin z)^{\frac{1}{i}} + (\cos z - \sqrt{-1}\,\sin z)^{\frac{1}{i}}}{2}$$

and

$$\frac{z}{i} = \frac{(\cos z + \sqrt{-1}\,\sin z)^{\frac{1}{i}} - (\cos z - \sqrt{-1}\,\sin z)^{\frac{1}{i}}}{2\sqrt{-1}}$$

[…]

[271]*§ 325*. If, however,[1397] one develops this product

$$(1+x)(1+x^2)(1+x^3)(1+x^4)(1+x^5)(1+x^6) \text{ \&c.},$$

the following series results:

$$1 + x + x^2 + 2x^3 + 2x^4 + 3x^5 + 4x^6 + 5x^7 + 6x^8 + 8x^9 + 10x^{10} + \text{ \&c.},$$

in which each coefficient indicates in how many ways the exponent of the corresponding [power of] x may be obtained from addition of unequal integer numbers. Thus the number 9 can be composed in 8 different ways by addition of unequal numbers, namely:

9 = 9	9 = 6+2+1
9 = 8+1	9 = 5+4
9 = 7+2	9 = 5+3+1
9 = 6+3	9 = 4+3+2